# Boot-Strapping in Simple Regression

Amal Agarwal, Sagar Chordia, Tuhin Sarkar

**Summary**

1. Bootstrap method can be used to improve stability of regression coefficient and reduce the length of confidence interval, when the overall error distribution is unknown or it is not the normal distribution. Primarily according to the different regression relationships, Bootstrap re-sampling methods can be divided into two types:
2. Model Based Bootstrap Regression-
3. Firstly, establish regression model with all samples and estimate the regression coefficients , .
4. Then re-sample the random residual and calculate the dependent variables, that is (X∗i , Y∗i ) in y∗i =βˆ0 + βˆ1xi + ε∗i , (i = 1, 2, · · · , n) is a model-based Bootstrap sample.
5. Cases Bootstrap Regression: The independent variable x and dependent variable y in correlation model regression are random variables and accord with joint distribution F(x, y). E(yi|xi) = f (xi) = β0 + β1xi, (i = 1, 2, · · · , n), (2) where β0, β1 are determined constants independent with X and n is the sample number. Assume yi = β0 + β1xi + εi, (i = 1, 2, · · , n), (3) where εi matches Gauss-Markov assumption, that is , matches equations (1), (2) and (3). Based on cases re-sampling in linear regression: Random select (x∗i , y∗i ) in original samples, i.e. cases Bootstrap sample.
6. Confidence Interval for Bootstrap Regression Coefficient β:

Since the error ε*i* and the distributions of β0 and β1 are unable to determine, it is difficult to get a statistic θ(*x*) whose distribution is known.

1. Estimation Method of Regression Coefficient β:

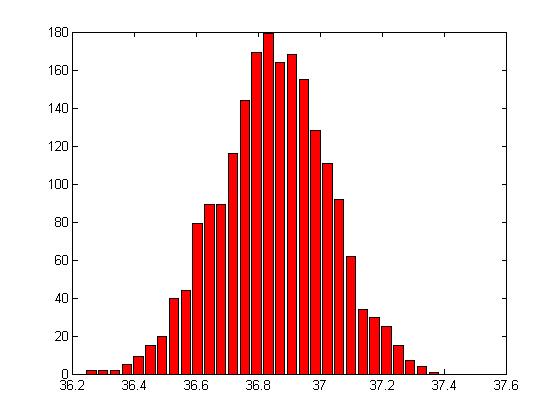
The following describes three common estimation methods of regression coefficient β, and combines them with Bootstrap method.

1. Least Squares Method Estimate(OLS):
2. Least Absolute Deviation Regression (LAD)
3. Least Median Squares Regression(LMS)

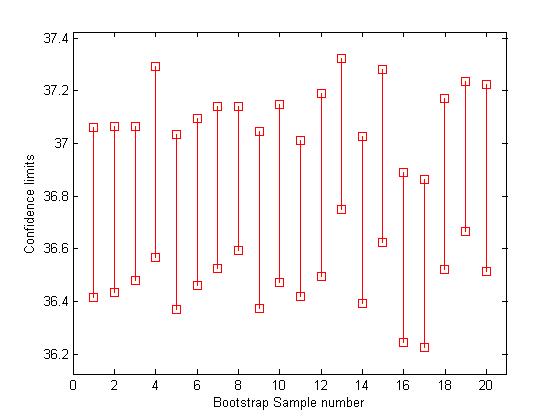
**Table 1 : Simulation regression results of Model-based error normal distribution**

***N*(0, 1), *B* = 2000, α = 0.05)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Statistic** | **OLS** | **LAD** | **LMS** |
| Confidence Interval | number containing | 734 | 1329 | 1524 |
| Lower Limit of Confidence Interval | minimum | 35.84046 | 35.54002 | 35.15892 |
| maximum | 37.02594 | 37.04596 | 37.3829 |
| mean | 36.51996 | 36.3844 | 36.33775 |
| median | 36.53213 | 36.40568 | 36.34371 |
| standard deviation | 0.179669 | 0.292927 | 0.346871 |
| Upper Limit of Confidence Interval | minimum | 36.62536 | 36.58277 | 35.15892 |
| maximum | 37.78652 | 38.06254 | 38.37167 |
| mean | 37.17265 | 37.30813 | 37.36648 |
| median | 37.16744 | 37.29341 | 37.36286 |
| standard deviation | 0.179017 | 0.274441 | 0.302602 |
|  | minimum | 36.24005 | 36.07642 | 35.74465 |
| maximum | 37.38269 | 37.51898 | 37.85468 |
| mean | 36.84631 | 36.84626 | 36.85212 |
| median | 36.84825 | 36.85237 | 36.85237 |
| standard deviation | 0.176028 | 0.27849 | 0.31962 |



**Figure 1. Histogram of coefficient estimation by OLS method for model-based Bootstrap model (error is normal distribution )**

****

**Figure 2. Confidence interval of coefficient estimation by OLS method for model-based Bootstrap model (error is normal distribution and *B* = 2000)**

**Table 2 : Simulation regression results of Model-based error uniform distribution**

**, *B* = 2000, α = 0.05)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **statistic** | **OLS** | **LAD** | **LMS** |
| Confidence Interval | number containing | 825 | 2000 | 1683 |
| Lower Limit of Confidence Interval | minimum | 36.82132 | 36.91626 | 36.62037 |
| maximum | 37.11288 | 37.29047 | 37.49834 |
| mean | 36.99008 | 37.09256 | 37.03786 |
| median | 36.99151 | 37.09249 | 37.03595 |
| standard deviation | 0.045735 | 0.057817 | 0.08292 |
| Upper Limit of Confidence Interval | minimum | 37.03052 | 36.85465 | 36.62037 |
| maximum | 37.32048 | 37.26026 | 37.57 |
| mean | 37.1553 | 37.0529 | 37.11343 |
| median | 37.15331 | 37.05355 | 37.11116 |
| standard deviation | 0.044622 | 0.062112 | 0.081627 |
|  | minimum | 36.92759 | 36.88545 | 36.66743 |
| maximum | 37.21668 | 37.27537 | 37.53417 |
| mean | 37.07269 | 37.07273 | 37.07564 |
| median | 37.07169 | 37.07306 | 37.07306 |
| standard deviation | 0.043174 | 0.059905 | 0.082197 |

**Table 3 : Cases simulation regression results (*B* = 2000, α = 0.05, ρ = 0.7)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Statistic** | **OLS** | **LAD** | **LMS** |
| Confidence Interval | Number containing | 511 | 1771 | 1833 |
| Lower Limit of Confidence Interval | minimum | -1.33659 | -1.80676 | -3.86731 |
| maximum | 1.121802 | 1.472863 | 1.48619 |
| mean | 0.472889 | 0.308815 | -1.07647 |
| median | 0.485913 | 0.383328 | -0.98229 |
| standard deviation | 0.226764 | 0.463216 | 0.661598 |
| Upper Limit of Confidence Interval | minimum | 0.36316 | 0.367907 | -3.86731 |
| maximum | 2.832343 | 3.414443 | 6.435961 |
| mean | 1.503245 | 1.63573 | 2.751256 |
| median | 1.481236 | 1.558748 | 2.70095 |
| standard deviation | 0.298448 | 0.451254 | 0.979869 |
|  | minimum | -0.4577 | -0.71943 | -1.66747 |
| maximum | 1.920027 | 2.274507 | 3.337119 |
| mean | 0.988067 | 0.972273 | 0.837392 |
| median | 0.978469 | 0.98243 | 0.952451 |
| standard deviation | 0.243123 | 0.431192 | 0.733461 |

**Code for Bootstrapping in Simple Regression**

**(To be run in Matlab version 7.10.0.499 (R2010a))**

1. **Model-based Bootstrap Regression**
2. ***Normal Error:***

% Model Based Bootstrap Regression Analysis with normal error

% Disclaimer: Wait for atleast 7 minutes for the program to give results

% Note the tables are automatically created as excel files in the MATLAB folder inside the documents (can be set while installation)

clear all;

clc;

n = 14; % sample size

B = 2000; % number of bootstrap samples

alpha = 0.05; % level of significance

X\_1 = ones(n,1);

X\_2 = 0+10\*rand(n,1);

X = [X\_1, X\_2];

% Original coefficients

beta0 = 0;

beta1 = 37;

% Original Model

er = normrnd(0,1,n,1);

Y = beta0 + beta1\*(X\_2) + er;

% regression on original data

[b\_OLS,bint,r,rint,stats] = regress(Y,X);

Y\_cap = b\_OLS(1) + b\_OLS(2)\*(X\_2);

sigma\_cap\_OLS = sqrt((sum((Y-(b\_OLS(1)+b\_OLS(2)\*X\_2)).^2))/(n-2));

% Bootstrap Samples

H = X\*(inv(X'\*X))\*X';

var\_r = zeros(n,1);

for i=1:n

var\_r(i) = stats(4)\*(1-H(i,i));

end

rs = r./var\_r; % Revised Studentized Residuals

rs\_r = rs - mean(rs); % Cntralized Residuals

f = @(x) x;

bootstat = bootstrp(B,f,rs\_r);

% b\_OLS\_b contains OLS estimated regression parameters in bootstrap samples

Y\_b = zeros(n,B);

b\_OLS\_b = zeros(2,B);

bint\_b = zeros(2,2,B);

r\_b = zeros(n,B);

rint\_b = zeros(n,2,B);

stats\_b = zeros(4,B);

sigma\_cap\_OLS\_b = zeros(1,B);

theta\_star\_OLS\_b = zeros(1,B);

for j=1:B

Y\_b(:,j) = b\_OLS(1) + b\_OLS(2)\*X\_2 + (bootstat(j,:))';

[b\_OLS\_b(:,j),bint\_b(:,:,j),r\_b(:,j),rint\_b(:,:,j),stats\_b(:,j)] = regress(Y\_b(:,j),X);

sigma\_cap\_OLS\_b(j) = sqrt((sum((Y\_b(:,j)-(b\_OLS(1)+b\_OLS(2)\*X\_2)).^2))/(n-2));

theta\_star\_OLS\_b(j) = (b\_OLS\_b(2,j)-(b\_OLS(2)))/((sigma\_cap\_OLS\_b(j))/(sqrt(sum((X\_2-mean(X\_2)).^2))));

end

% histogram plot of beta1 estimated by OLS

[ne,xc] = hist(b\_OLS\_b(2,:),30,'-r');

bh = bar(xc,ne);

set(bh,'facecolor',[1 0 0]);

% Analysis on Beta1 estimate by OLS

min\_b1\_OLS\_b = min(b\_OLS\_b(2,:));

max\_b1\_OLS\_b = max(b\_OLS\_b(2,:));

mean\_b1\_OLS\_b = mean(b\_OLS\_b(2,:));

median\_b1\_OLS\_b = median(b\_OLS\_b(2,:));

std\_b1\_OLS\_b = std(b\_OLS\_b(2,:));

% Confidence Intervals for beta1 using OLS estimated beta1

theta\_star\_OLS\_b\_sorted = sort(theta\_star\_OLS\_b);

omega\_OLS = theta\_star\_OLS\_b\_sorted((1-(alpha/2))\*B);

% In original sample

CI\_L\_beta1\_OLS = b\_OLS(2)-((omega\_OLS\*sigma\_cap\_OLS)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

CI\_U\_beta1\_OLS = b\_OLS(2)+((omega\_OLS\*sigma\_cap\_OLS)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

% In Bootstrap samples

CI\_L\_beta1\_OLS\_b = b\_OLS\_b(2,:)-((omega\_OLS\*sigma\_cap\_OLS\_b)./(sqrt(sum((X\_2-mean(X\_2)).^2))));

CI\_U\_beta1\_OLS\_b = b\_OLS\_b(2,:)+((omega\_OLS\*sigma\_cap\_OLS\_b)./(sqrt(sum((X\_2-mean(X\_2)).^2))));

% Analysis on Confidence Intervals for beta1 estimated by OLS

% CI Lower Limit

min\_CI\_L\_beta1\_OLS\_b = min(CI\_L\_beta1\_OLS\_b);

max\_CI\_L\_beta1\_OLS\_b = max(CI\_L\_beta1\_OLS\_b);

mean\_CI\_L\_beta1\_OLS\_b = mean(CI\_L\_beta1\_OLS\_b);

median\_CI\_L\_beta1\_OLS\_b = median(CI\_L\_beta1\_OLS\_b);

std\_CI\_L\_beta1\_OLS\_b = std(CI\_L\_beta1\_OLS\_b);

% CI Upper Limit

min\_CI\_U\_beta1\_OLS\_b = min(CI\_U\_beta1\_OLS\_b);

max\_CI\_U\_beta1\_OLS\_b = max(CI\_U\_beta1\_OLS\_b);

mean\_CI\_U\_beta1\_OLS\_b = mean(CI\_U\_beta1\_OLS\_b);

median\_CI\_U\_beta1\_OLS\_b = median(CI\_U\_beta1\_OLS\_b);

std\_CI\_U\_beta1\_OLS\_b = std(CI\_U\_beta1\_OLS\_b);

% Bootstrap sample number for which beta1 lies in the confidence limits by OLS

diff\_OLS = Inf\*ones(1,B);

for j=1:B

if beta1<=CI\_U\_beta1\_OLS\_b(j) && beta1>=CI\_L\_beta1\_OLS\_b(j)

diff\_OLS(j) = abs(((CI\_L\_beta1\_OLS\_b(j)+CI\_U\_beta1\_OLS\_b(j))/2)-beta1);

end

end

for j=1:B

if diff\_OLS(j)==min(diff\_OLS)

j\_min=j;

end

end

beta1\_CI\_check\_OLS = j\_min;

% Confidence Intervals plot for beta1 by OLS

B\_plot = [(1:20)',(1:20)'];

y1=min(CI\_L\_beta1\_OLS\_b(1:20))-0.1;

y2=max(CI\_U\_beta1\_OLS\_b(1:20))+0.1;

CI\_plot = [CI\_L\_beta1\_OLS\_b(1:20)',CI\_U\_beta1\_OLS\_b(1:20)'];

figure, plot(B\_plot(1,:), CI\_plot(1,:), '-rs',...

B\_plot(2,:), CI\_plot(2,:), '-rs',...

B\_plot(3,:), CI\_plot(3,:), '-rs',...

B\_plot(4,:), CI\_plot(4,:), '-rs',...

B\_plot(5,:), CI\_plot(5,:), '-rs',...

B\_plot(6,:), CI\_plot(6,:), '-rs',...

B\_plot(7,:), CI\_plot(7,:), '-rs',...

B\_plot(8,:), CI\_plot(8,:), '-rs',...

B\_plot(9,:), CI\_plot(9,:), '-rs',...

B\_plot(10,:), CI\_plot(10,:), '-rs',...

B\_plot(11,:), CI\_plot(11,:), '-rs',...

B\_plot(12,:), CI\_plot(12,:), '-rs',...

B\_plot(13,:), CI\_plot(13,:), '-rs',...

B\_plot(14,:), CI\_plot(14,:), '-rs',...

B\_plot(15,:), CI\_plot(15,:), '-rs',...

B\_plot(16,:), CI\_plot(16,:), '-rs',...

B\_plot(17,:), CI\_plot(17,:), '-rs',...

B\_plot(18,:), CI\_plot(18,:), '-rs',...

B\_plot(19,:), CI\_plot(19,:), '-rs',...

B\_plot(20,:), CI\_plot(20,:), '-rs');

title ('Confidence interval of coefficient Beta1 estimation by OLS method for model-based Bootstrap model(error is normal distribution and B = 2000)');

axis([0 21 y1 y2]);

xlabel('Bootstrap Sample number');

ylabel('Confidence limits');

% LAD estimation on original sample

beta0\_LAD = zeros(n,n);

beta1\_LAD = zeros(n,n);

d = zeros(n,n);

for i = 1:n

for j = 1:n

if i~=j

beta1\_LAD(i,j) = (Y(j)-Y(i))/(X\_2(j)-X\_2(i));

beta0\_LAD(i,j) = Y(j)-beta1\_LAD(i,j)\*X\_2(j);

d(i,j) = sum(abs(Y-(beta0\_LAD(i,j)+beta1\_LAD(i,j)\*X\_2)));

end

end

end

d\_min = Inf;

for i=1:n

for j=1:n

if i~=j

if d\_min>d(i,j);

d\_min=d(i,j);

i\_min = i;

j\_min = j;

end

end

end

end

b0\_LAD = beta0\_LAD(i\_min,j\_min);

b1\_LAD = beta1\_LAD(i\_min,j\_min);

% LAD estimation on Bootstrap Samples

beta0\_LAD\_b = zeros(n,n,B);

beta1\_LAD\_b = zeros(n,n,B);

d\_b = zeros(n,n,B);

for k = 1:B

for i = 1:n

for j = 1:n

if i~=j

beta1\_LAD\_b(i,j,k) = (Y\_b(j,k)-Y\_b(i,k))/(X\_2(j)-X\_2(i));

beta0\_LAD\_b(i,j,k) = Y\_b(j,k)-beta1\_LAD\_b(i,j,k)\*X\_2(j);

d(i,j,k) = sum(abs(Y\_b(:,k)-(beta0\_LAD\_b(i,j,k)+beta1\_LAD\_b(i,j,k)\*X\_2)));

end

end

end

end

d\_min = Inf\*ones(1,B);

i\_min = zeros(1,B);

j\_min = zeros(1,B);

for k=1:B

for i=1:n

for j=1:n

if i~=j

if d\_min(k)>d(i,j,k);

d\_min(k) = d(i,j,k);

i\_min(k) = i;

j\_min(k) = j;

end

end

end

end

end

b0\_LAD\_b = zeros(1,B);

b1\_LAD\_b = zeros(1,B);

for k=1:B

b0\_LAD\_b(k) = beta0\_LAD\_b(i\_min(k),j\_min(k),k);

b1\_LAD\_b(k) = beta1\_LAD\_b(i\_min(k),j\_min(k),k);

end

% Analysis on Beta1 estimate by LAD

min\_b1\_LAD\_b = min(b1\_LAD\_b(1,:));

max\_b1\_LAD\_b = max(b1\_LAD\_b(1,:));

mean\_b1\_LAD\_b = mean(b1\_LAD\_b(1,:));

median\_b1\_LAD\_b = median(b1\_LAD\_b(1,:));

std\_b1\_LAD\_b = std(b1\_LAD\_b(1,:));

% Confidence Intervals for beta1 using LAD estimated beta1

sigma\_cap\_LAD = sqrt((sum((Y-(b0\_LAD+b1\_LAD\*X\_2)).^2))/(n-2));

sigma\_cap\_LAD\_b = zeros(1,B);

theta\_star\_LAD\_b = zeros(1,B);

for j=1:B

sigma\_cap\_LAD\_b(j) = sqrt((sum((Y\_b(:,j)-(b0\_LAD+b1\_LAD\*X\_2)).^2))/(n-2));

theta\_star\_LAD\_b(j) = (b1\_LAD\_b(j)-(b1\_LAD))/((sigma\_cap\_LAD\_b(j))/(sqrt(sum((X\_2-mean(X\_2)).^2))));

end

theta\_star\_LAD\_b\_sorted = sort(theta\_star\_LAD\_b);

omega\_LAD = theta\_star\_LAD\_b\_sorted((1-(alpha/2))\*B);

% In original sample

CI\_L\_beta1\_LAD = b1\_LAD-((omega\_LAD\*sigma\_cap\_LAD)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

CI\_U\_beta1\_LAD = b1\_LAD+((omega\_LAD\*sigma\_cap\_LAD)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

% In Bootstrap samples

CI\_L\_beta1\_LAD\_b = b1\_LAD\_b(1,:)-((omega\_LAD\*sigma\_cap\_LAD\_b)./(sqrt(sum((X\_2-mean(X\_2)).^2))));

CI\_U\_beta1\_LAD\_b = b1\_LAD\_b(1,:)+((omega\_LAD\*sigma\_cap\_LAD\_b)./(sqrt(sum((X\_2-mean(X\_2)).^2))));

% Analysis on Confidence Intervals for beta1 estimated by LAD

% CI Lower Limit

min\_CI\_L\_beta1\_LAD\_b = min(CI\_L\_beta1\_LAD\_b);

max\_CI\_L\_beta1\_LAD\_b = max(CI\_L\_beta1\_LAD\_b);

mean\_CI\_L\_beta1\_LAD\_b = mean(CI\_L\_beta1\_LAD\_b);

median\_CI\_L\_beta1\_LAD\_b = median(CI\_L\_beta1\_LAD\_b);

std\_CI\_L\_beta1\_LAD\_b = std(CI\_L\_beta1\_LAD\_b);

% CI Upper Limit

min\_CI\_U\_beta1\_LAD\_b = min(CI\_U\_beta1\_LAD\_b);

max\_CI\_U\_beta1\_LAD\_b = max(CI\_U\_beta1\_LAD\_b);

mean\_CI\_U\_beta1\_LAD\_b = mean(CI\_U\_beta1\_LAD\_b);

median\_CI\_U\_beta1\_LAD\_b = median(CI\_U\_beta1\_LAD\_b);

std\_CI\_U\_beta1\_LAD\_b = std(CI\_U\_beta1\_LAD\_b);

% Bootstrap sample number for which beta1 lies in the confidence limits by LAD

diff\_LAD = Inf\*ones(1,B);

for j=1:B

if beta1<=CI\_U\_beta1\_LAD\_b(j) && beta1>=CI\_L\_beta1\_LAD\_b(j)

diff\_LAD(j) = abs(((CI\_L\_beta1\_LAD\_b(j)+CI\_U\_beta1\_LAD\_b(j))/2)-beta1);

end

end

for j=1:B

if diff\_LAD(j)==min(diff\_LAD)

j\_min=j;

end

end

beta1\_CI\_check\_LAD = j\_min;

% LMS estimation on original sample

d\_cap = Inf;

% X\_2\_sorted = sort(X\_2);

beta0\_LMS = zeros(n,n,n);

beta1\_LMS = zeros(n,n,n);

d = zeros(n,n,n);

for i=1:n

for j=1:n

for k=1:n

beta1\_LMS(i,j,k) = (Y(i)-Y(k))/(X\_2(i)-X\_2(k));

beta0\_LMS(i,j,k) = Y(j)+Y(k)-beta1\_LMS(i,j,k)\*(X\_2(j)+X\_2(k));

d(i,j,k) = median((Y-(beta0\_LMS(i,j,k)+beta1\_LMS(i,j,k)\*X\_2)).^2);

end

end

end

for i=1:n

for j=1:n

for k=1:n

if d\_cap>d(i,j,k);

d\_cap=d(i,j,k);

i\_cap = i;

j\_cap = j;

k\_cap = k;

end

end

end

end

b0\_LMS = beta0\_LMS(i\_cap,j\_cap,k\_cap);

b1\_LMS = beta1\_LMS(i\_cap,j\_cap,k\_cap);

% LMS estimation on Bootstrap Samples

d\_cap\_b = Inf\*ones(1,B);

X\_2\_sorted = sort(X\_2);

beta0\_LMS\_b = zeros(n,n,n,B);

beta1\_LMS\_b = zeros(n,n,n,B);

d = zeros(n,n,n,B);

for l = 1:B

for i = 1:n

for j = 1:n

for k=1:n

beta1\_LMS\_b(i,j,k,l) = (Y\_b(i,l)-Y\_b(k,l))/(X\_2(i)-X\_2(k));

beta0\_LMS\_b(i,j,k,l) = Y\_b(j,l)+Y\_b(k,l)-beta1\_LMS\_b(i,j,k,l)\*(X\_2(j)+X\_2(k));

d(i,j,k,l) = median((Y\_b(:,l)-(beta0\_LMS\_b(i,j,k,l)+beta1\_LMS\_b(i,j,k,l)\*X\_2)).^2);

end

end

end

end

i\_cap\_b = zeros(1,B);

j\_cap\_b = zeros(1,B);

k\_cap\_b = zeros(1,B);

for l=1:B

for i=1:n

for j=1:n

for k=1:n

if d\_cap\_b(l)>d(i,j,k,l);

d\_cap\_b(l) = d(i,j,k,l);

i\_cap\_b(l) = i;

j\_cap\_b(l) = j;

k\_cap\_b(l) = k;

end

end

end

end

end

b0\_LMS\_b = zeros(1,B);

b1\_LMS\_b = zeros(1,B);

for l=1:B

b0\_LMS\_b(l) = beta0\_LMS\_b(i\_cap\_b(l),j\_cap\_b(l),k\_cap\_b(l),l);

b1\_LMS\_b(l) = beta1\_LMS\_b(i\_cap\_b(l),j\_cap\_b(l),k\_cap\_b(l),l);

end

% Analysis on Beta1 estimate by LMS

min\_b1\_LMS\_b = min(b1\_LMS\_b(1,:));

max\_b1\_LMS\_b = max(b1\_LMS\_b(1,:));

mean\_b1\_LMS\_b = mean(b1\_LMS\_b(1,:));

median\_b1\_LMS\_b = median(b1\_LMS\_b(1,:));

std\_b1\_LMS\_b = std(b1\_LMS\_b(1,:));

% Confidence Intervals for beta1 using LMS estimated beta1

sigma\_cap\_LMS = sqrt((sum((Y-(b0\_LMS+b1\_LMS\*X\_2)).^2))/(n-2));

sigma\_cap\_LMS\_b = zeros(1,B);

theta\_star\_LMS\_b = zeros(1,B);

for j=1:B

sigma\_cap\_LMS\_b(j) = sqrt((sum((Y\_b(:,j)-(b0\_LMS+b1\_LMS\*X\_2)).^2))/(n-2));

theta\_star\_LMS\_b(j) = (b1\_LMS\_b(j)-(b1\_LMS))/((sigma\_cap\_LMS\_b(j))/(sqrt(sum((X\_2-mean(X\_2)).^2))));

end

theta\_star\_LMS\_b\_sorted = sort(theta\_star\_LMS\_b);

omega\_LMS = theta\_star\_LMS\_b\_sorted((1-(alpha/2))\*B);

% In original sample

CI\_L\_beta1\_LMS = b1\_LMS-((omega\_LMS\*sigma\_cap\_LMS)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

CI\_U\_beta1\_LMS = b1\_LMS+((omega\_LMS\*sigma\_cap\_LMS)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

% In Bootstrap samples

CI\_L\_beta1\_LMS\_b = b1\_LMS\_b(1,:)-((omega\_LMS\*sigma\_cap\_LMS\_b)./(sqrt(sum((X\_2-mean(X\_2)).^2))));

CI\_U\_beta1\_LMS\_b = b1\_LMS\_b(1,:)+((omega\_LMS\*sigma\_cap\_LMS\_b)./(sqrt(sum((X\_2-mean(X\_2)).^2))));

% Analysis on Confidence Intervals for beta1 estimated by LMS

% CI Lower Limit

min\_CI\_L\_beta1\_LMS\_b = min(CI\_L\_beta1\_LMS\_b);

max\_CI\_L\_beta1\_LMS\_b = max(CI\_L\_beta1\_LMS\_b);

mean\_CI\_L\_beta1\_LMS\_b = mean(CI\_L\_beta1\_LMS\_b);

median\_CI\_L\_beta1\_LMS\_b = median(CI\_L\_beta1\_LMS\_b);

std\_CI\_L\_beta1\_LMS\_b = std(CI\_L\_beta1\_LMS\_b);

% CI Upper Limit

min\_CI\_U\_beta1\_LMS\_b = min(CI\_U\_beta1\_LMS\_b);

max\_CI\_U\_beta1\_LMS\_b = max(CI\_U\_beta1\_LMS\_b);

mean\_CI\_U\_beta1\_LMS\_b = mean(CI\_U\_beta1\_LMS\_b);

median\_CI\_U\_beta1\_LMS\_b = median(CI\_U\_beta1\_LMS\_b);

std\_CI\_U\_beta1\_LMS\_b = std(CI\_U\_beta1\_LMS\_b);

% Bootstrap sample number for which beta1 lies in the confidence limits by LMS

diff\_LMS = Inf\*ones(1,B);

for j=1:B

if beta1<=CI\_U\_beta1\_LMS\_b(j) && beta1>=CI\_L\_beta1\_LMS\_b(j)

diff\_LMS(j) = abs(((CI\_L\_beta1\_LMS\_b(j)+CI\_U\_beta1\_LMS\_b(j))/2)-beta1);

end

end

for j=1:B

if diff\_LMS(j)==min(diff\_LMS)

j\_min=j;

end

end

beta1\_CI\_check\_LMS = j\_min;

% generating table in excel

filename = 'table1.xlsx';

A = {'Beta\_1','statistic','OLS','LAD','LMS';...

'Confidence Interval','Number containing beta1',...

beta1\_CI\_check\_OLS,beta1\_CI\_check\_LAD,beta1\_CI\_check\_LMS;...

'Lower Limit of Confidence Interval',...

'minimum',min\_CI\_L\_beta1\_OLS\_b,min\_CI\_L\_beta1\_LAD\_b,min\_CI\_L\_beta1\_LMS\_b;...

'','maximum',max\_CI\_L\_beta1\_OLS\_b,max\_CI\_L\_beta1\_LAD\_b,max\_CI\_L\_beta1\_LMS\_b;...

'','mean',mean\_CI\_L\_beta1\_OLS\_b,mean\_CI\_L\_beta1\_LAD\_b,mean\_CI\_L\_beta1\_LMS\_b;...

'','median',median\_CI\_L\_beta1\_OLS\_b,median\_CI\_L\_beta1\_LAD\_b,median\_CI\_L\_beta1\_LMS\_b;...

'','standard deviation',std\_CI\_L\_beta1\_OLS\_b,std\_CI\_L\_beta1\_LAD\_b,std\_CI\_L\_beta1\_LMS\_b;

'Upper Limit of Confidence Interval',...

'minimum',min\_CI\_U\_beta1\_OLS\_b,min\_CI\_U\_beta1\_LAD\_b,min\_CI\_L\_beta1\_LMS\_b;...

'','maximum',max\_CI\_U\_beta1\_OLS\_b,max\_CI\_U\_beta1\_LAD\_b,max\_CI\_U\_beta1\_LMS\_b;...

'','mean',mean\_CI\_U\_beta1\_OLS\_b,mean\_CI\_U\_beta1\_LAD\_b,mean\_CI\_U\_beta1\_LMS\_b;...

'','median',median\_CI\_U\_beta1\_OLS\_b,median\_CI\_U\_beta1\_LAD\_b,median\_CI\_U\_beta1\_LMS\_b;...

'','standard deviation',std\_CI\_U\_beta1\_OLS\_b,std\_CI\_U\_beta1\_LAD\_b,std\_CI\_U\_beta1\_LMS\_b;

'Estimated Beta\_1',...

'minimum',min\_b1\_OLS\_b,min\_b1\_LAD\_b,min\_b1\_LMS\_b;...

'','maximum',max\_b1\_OLS\_b,max\_b1\_LAD\_b,max\_b1\_LMS\_b;...

'','mean',mean\_b1\_OLS\_b,mean\_b1\_LAD\_b,mean\_b1\_LMS\_b;...

'','median',median\_b1\_OLS\_b,median\_b1\_LAD\_b,median\_b1\_LMS\_b;...

'','standard deviation',std\_b1\_OLS\_b,std\_b1\_LAD\_b,std\_b1\_LMS\_b;};

sheet = 1;

xlRange = 'A1';

xlswrite(filename,A,sheet,xlRange);

1. ***Uniform Error:***

**Same code as above except replace the “original model” block as:**

% Original Model

er = -sqrt(12)+2\*sqrt(12)\*rand(n,1);

Y = beta0 + beta1\*(X\_2) + er;

1. **Cases Bootstrap Regression**

% Cases Bootstrap Regression Analysis

% Disclaimer: Wait for atleast 7 minutes for the program to give results

% Note: The tables are automatically created as excel files in the MATLAB folder inside the documents (can be set while installation)

clear all;

clc;

n = 14; % sample size

B = 2000; % number of bootstrap samples

alpha = 0.05; % level of significance

% Original coefficients

beta0 = 1;

beta1 = 0.7;

% Original Model

mu = [0,1];

SIGMA = [1, 0.7; 0.7, 1];

xy = mvnrnd(mu,SIGMA,n);

X\_1 = ones(n,1);

X\_2 = xy(:,1);

X = [X\_1, X\_2];

Y = xy(:,2);

% regression on original data

[b\_OLS,bint,r,rint,stats] = regress(Y,X);

Y\_cap = b\_OLS(1) + b\_OLS(2)\*(X\_2);

sigma\_cap\_OLS = sqrt((sum((Y-(b\_OLS(1)+b\_OLS(2)\*X\_2)).^2))/(n-2));

% Bootstrap Samples

f = @(x) x;

bootstat\_xy = bootstrp(B,f,xy);

% b\_OLS\_b contains OLS estimated regression parameters in bootstrap samples

X\_2\_b = zeros(n,B);

X\_b = zeros(n,2,B);

Y\_b = zeros(n,B);

b\_OLS\_b = zeros(2,B);

bint\_b = zeros(2,2,B);

r\_b = zeros(n,B);

rint\_b = zeros(n,2,B);

stats\_b = zeros(4,B);

sigma\_cap\_OLS\_b = zeros(1,B);

theta\_star\_OLS\_b = zeros(1,B);

for j=1:B

X\_2\_b(:,j) = (bootstat\_xy(j,1:n))';

X\_b(:,:,j) = [X\_1, X\_2\_b(:,j)];

Y\_b(:,j) = (bootstat\_xy(j,n+1:2\*n))';

[b\_OLS\_b(:,j),bint\_b(:,:,j),r\_b(:,j),rint\_b(:,:,j),stats\_b(:,j)] = regress(Y\_b(:,j),X\_b(:,:,j));

sigma\_cap\_OLS\_b(j) = sqrt((sum((Y\_b(:,j)-(b\_OLS(1)+b\_OLS(2)\*X\_2)).^2))/(n-2));

theta\_star\_OLS\_b(j) = (b\_OLS\_b(2,j)-(b\_OLS(2)))/((sigma\_cap\_OLS\_b(j))/(sqrt(sum((X\_2-mean(X\_2)).^2))));

end

% histogram plot of beta1 estimated by OLS

[ne,xc] = hist(b\_OLS\_b(2,:),30,'-r');

bh = bar(xc,ne);

set(bh,'facecolor',[1 0 0]);

% Analysis on Beta1 estimate by OLS

min\_b1\_OLS\_b = min(b\_OLS\_b(2,:));

max\_b1\_OLS\_b = max(b\_OLS\_b(2,:));

mean\_b1\_OLS\_b = mean(b\_OLS\_b(2,:));

median\_b1\_OLS\_b = median(b\_OLS\_b(2,:));

std\_b1\_OLS\_b = std(b\_OLS\_b(2,:));

% Confidence Intervals for beta1 using OLS estimated beta1

theta\_star\_OLS\_b\_sorted = sort(theta\_star\_OLS\_b);

omega\_OLS = theta\_star\_OLS\_b\_sorted((1-(alpha/2))\*B);

% In original sample

CI\_L\_beta1\_OLS = b\_OLS(2)-((omega\_OLS\*sigma\_cap\_OLS)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

CI\_U\_beta1\_OLS = b\_OLS(2)+((omega\_OLS\*sigma\_cap\_OLS)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

% In Bootstrap samples

CI\_L\_beta1\_OLS\_b = zeros(1,B);

CI\_U\_beta1\_OLS\_b = zeros(1,B);

for j=1:B

CI\_L\_beta1\_OLS\_b(j) = b\_OLS\_b(2,j)-((omega\_OLS\*sigma\_cap\_OLS\_b(j))./(sqrt(sum((X\_2\_b(:,j)-mean(X\_2\_b(:,j))).^2))));

CI\_U\_beta1\_OLS\_b(j) = b\_OLS\_b(2,j)+((omega\_OLS\*sigma\_cap\_OLS\_b(j))./(sqrt(sum((X\_2\_b(:,j)-mean(X\_2\_b(:,j))).^2))));

end

% Analysis on Confidence Intervals for beta1 estimated by OLS

% CI Lower Limit

min\_CI\_L\_beta1\_OLS\_b = min(CI\_L\_beta1\_OLS\_b);

max\_CI\_L\_beta1\_OLS\_b = max(CI\_L\_beta1\_OLS\_b);

mean\_CI\_L\_beta1\_OLS\_b = mean(CI\_L\_beta1\_OLS\_b);

median\_CI\_L\_beta1\_OLS\_b = median(CI\_L\_beta1\_OLS\_b);

std\_CI\_L\_beta1\_OLS\_b = std(CI\_L\_beta1\_OLS\_b);

% CI Upper Limit

min\_CI\_U\_beta1\_OLS\_b = min(CI\_U\_beta1\_OLS\_b);

max\_CI\_U\_beta1\_OLS\_b = max(CI\_U\_beta1\_OLS\_b);

mean\_CI\_U\_beta1\_OLS\_b = mean(CI\_U\_beta1\_OLS\_b);

median\_CI\_U\_beta1\_OLS\_b = median(CI\_U\_beta1\_OLS\_b);

std\_CI\_U\_beta1\_OLS\_b = std(CI\_U\_beta1\_OLS\_b);

% Bootstrap sample number for which beta1 lies in the confidence limits by OLS

diff\_OLS = Inf\*ones(1,B);

for j=1:B

if beta1<=CI\_U\_beta1\_OLS\_b(j) && beta1>=CI\_L\_beta1\_OLS\_b(j)

diff\_OLS(j) = abs(((CI\_L\_beta1\_OLS\_b(j)+CI\_U\_beta1\_OLS\_b(j))/2)-beta1);

end

end

for j=1:B

if diff\_OLS(j)==min(diff\_OLS)

j\_min=j;

end

end

beta1\_CI\_check\_OLS = j\_min;

% Confidence Intervals plot for beta1 by OLS

B\_plot = [(1:20)',(1:20)'];

y1=min(CI\_L\_beta1\_OLS\_b(1:20))-0.1;

y2=max(CI\_U\_beta1\_OLS\_b(1:20))+0.1;

CI\_plot = [CI\_L\_beta1\_OLS\_b(1:20)',CI\_U\_beta1\_OLS\_b(1:20)'];

figure, plot(B\_plot(1,:), CI\_plot(1,:), '-rs',...

B\_plot(2,:), CI\_plot(2,:), '-rs',...

B\_plot(3,:), CI\_plot(3,:), '-rs',...

B\_plot(4,:), CI\_plot(4,:), '-rs',...

B\_plot(5,:), CI\_plot(5,:), '-rs',...

B\_plot(6,:), CI\_plot(6,:), '-rs',...

B\_plot(7,:), CI\_plot(7,:), '-rs',...

B\_plot(8,:), CI\_plot(8,:), '-rs',...

B\_plot(9,:), CI\_plot(9,:), '-rs',...

B\_plot(10,:), CI\_plot(10,:), '-rs',...

B\_plot(11,:), CI\_plot(11,:), '-rs',...

B\_plot(12,:), CI\_plot(12,:), '-rs',...

B\_plot(13,:), CI\_plot(13,:), '-rs',...

B\_plot(14,:), CI\_plot(14,:), '-rs',...

B\_plot(15,:), CI\_plot(15,:), '-rs',...

B\_plot(16,:), CI\_plot(16,:), '-rs',...

B\_plot(17,:), CI\_plot(17,:), '-rs',...

B\_plot(18,:), CI\_plot(18,:), '-rs',...

B\_plot(19,:), CI\_plot(19,:), '-rs',...

B\_plot(20,:), CI\_plot(20,:), '-rs');

title ('Confidence interval of coefficient Beta1 estimation by OLS method for model-based Bootstrap model(error is normal distribution and B = 2000)');

axis([0 21 y1 y2]);

xlabel('Bootstrap Sample number');

ylabel('Confidence limits');

% LAD estimation on original sample

beta0\_LAD = zeros(n,n);

beta1\_LAD = zeros(n,n);

d = zeros(n,n);

for i = 1:n

for j = 1:n

if i~=j

beta1\_LAD(i,j) = (Y(j)-Y(i))/(X\_2(j)-X\_2(i));

beta0\_LAD(i,j) = Y(j)-beta1\_LAD(i,j)\*X\_2(j);

d(i,j) = sum(abs(Y-(beta0\_LAD(i,j)+beta1\_LAD(i,j)\*X\_2)));

end

end

end

d\_min = Inf;

for i=1:n

for j=1:n

if i~=j

if d\_min>d(i,j);

d\_min=d(i,j);

i\_min = i;

j\_min = j;

end

end

end

end

b0\_LAD = beta0\_LAD(i\_min,j\_min);

b1\_LAD = beta1\_LAD(i\_min,j\_min);

% LAD estimation on Bootstrap Samples

beta0\_LAD\_b = zeros(n,n,B);

beta1\_LAD\_b = zeros(n,n,B);

d\_b = zeros(n,n,B);

for k = 1:B

for i = 1:n

for j = 1:n

if i~=j

beta1\_LAD\_b(i,j,k) = (Y\_b(j,k)-Y\_b(i,k))/(X\_2\_b(j,k)-X\_2\_b(i,k));

beta0\_LAD\_b(i,j,k) = Y\_b(j,k)-beta1\_LAD\_b(i,j,k)\*X\_2\_b(j,k);

d(i,j,k) = sum(abs(Y\_b(:,k)-(beta0\_LAD\_b(i,j,k)+beta1\_LAD\_b(i,j,k)\*X\_2\_b(:,k))));

end

end

end

end

d\_min = Inf\*ones(1,B);

i\_min = zeros(1,B);

j\_min = zeros(1,B);

for k=1:B

for i=1:n

for j=1:n

if i~=j

if d\_min(k)>d(i,j,k);

d\_min(k) = d(i,j,k);

i\_min(k) = i;

j\_min(k) = j;

end

end

end

end

end

b0\_LAD\_b = zeros(1,B);

b1\_LAD\_b = zeros(1,B);

for k=1:B

b0\_LAD\_b(k) = beta0\_LAD\_b(i\_min(k),j\_min(k),k);

b1\_LAD\_b(k) = beta1\_LAD\_b(i\_min(k),j\_min(k),k);

end

% Analysis on Beta1 estimate by LAD

min\_b1\_LAD\_b = min(b1\_LAD\_b(1,:));

max\_b1\_LAD\_b = max(b1\_LAD\_b(1,:));

mean\_b1\_LAD\_b = mean(b1\_LAD\_b(1,:));

median\_b1\_LAD\_b = median(b1\_LAD\_b(1,:));

std\_b1\_LAD\_b = std(b1\_LAD\_b(1,:));

% Confidence Intervals for beta1 using LAD estimated beta1

sigma\_cap\_LAD = sqrt((sum((Y-(b0\_LAD+b1\_LAD\*X\_2)).^2))/(n-2));

sigma\_cap\_LAD\_b = zeros(1,B);

theta\_star\_LAD\_b = zeros(1,B);

for j=1:B

sigma\_cap\_LAD\_b(j) = sqrt((sum((Y\_b(:,j)-(b0\_LAD+b1\_LAD\*X\_2\_b(:,j))).^2))/(n-2));

theta\_star\_LAD\_b(j) = (b1\_LAD\_b(j)-(b1\_LAD))/((sigma\_cap\_LAD\_b(j))/(sqrt(sum((X\_2\_b(:,j)-mean(X\_2\_b(:,j))).^2))));

end

theta\_star\_LAD\_b\_sorted = sort(theta\_star\_LAD\_b);

omega\_LAD = theta\_star\_LAD\_b\_sorted((1-(alpha/2))\*B);

% In original sample

CI\_L\_beta1\_LAD = b1\_LAD-((omega\_LAD\*sigma\_cap\_LAD)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

CI\_U\_beta1\_LAD = b1\_LAD+((omega\_LAD\*sigma\_cap\_LAD)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

% In Bootstrap samples

CI\_L\_beta1\_LAD\_b = zeros(1,B);

CI\_U\_beta1\_LAD\_b = zeros(1,B);

for j=1:B

CI\_L\_beta1\_LAD\_b(j) = b1\_LAD\_b(1,j)-((omega\_LAD\*sigma\_cap\_LAD\_b(j))./(sqrt(sum((X\_2\_b(:,j)-mean(X\_2\_b(:,j))).^2))));

CI\_U\_beta1\_LAD\_b(j) = b1\_LAD\_b(1,j)+((omega\_LAD\*sigma\_cap\_LAD\_b(j))./(sqrt(sum((X\_2\_b(:,j)-mean(X\_2\_b(:,j))).^2))));

end

% Analysis on Confidence Intervals for beta1 estimated by LAD

% CI Lower Limit

min\_CI\_L\_beta1\_LAD\_b = min(CI\_L\_beta1\_LAD\_b);

max\_CI\_L\_beta1\_LAD\_b = max(CI\_L\_beta1\_LAD\_b);

mean\_CI\_L\_beta1\_LAD\_b = mean(CI\_L\_beta1\_LAD\_b);

median\_CI\_L\_beta1\_LAD\_b = median(CI\_L\_beta1\_LAD\_b);

std\_CI\_L\_beta1\_LAD\_b = std(CI\_L\_beta1\_LAD\_b);

% CI Upper Limit

min\_CI\_U\_beta1\_LAD\_b = min(CI\_U\_beta1\_LAD\_b);

max\_CI\_U\_beta1\_LAD\_b = max(CI\_U\_beta1\_LAD\_b);

mean\_CI\_U\_beta1\_LAD\_b = mean(CI\_U\_beta1\_LAD\_b);

median\_CI\_U\_beta1\_LAD\_b = median(CI\_U\_beta1\_LAD\_b);

std\_CI\_U\_beta1\_LAD\_b = std(CI\_U\_beta1\_LAD\_b);

% Bootstrap sample number for which beta1 lies in the confidence limits by LAD

diff\_LAD = Inf\*ones(1,B);

for j=1:B

if beta1<=CI\_U\_beta1\_LAD\_b(j) && beta1>=CI\_L\_beta1\_LAD\_b(j)

diff\_LAD(j) = abs(((CI\_L\_beta1\_LAD\_b(j)+CI\_U\_beta1\_LAD\_b(j))/2)-beta1);

end

end

for j=1:B

if diff\_LAD(j)==min(diff\_LAD)

j\_min=j;

end

end

beta1\_CI\_check\_LAD = j\_min;

% LMS estimation on original sample

d\_cap = Inf;

% X\_2\_sorted = sort(X\_2);

beta0\_LMS = zeros(n,n,n);

beta1\_LMS = zeros(n,n,n);

d = zeros(n,n,n);

for i=1:n

for j=1:n

for k=1:n

beta1\_LMS(i,j,k) = (Y(i)-Y(k))/(X\_2(i)-X\_2(k));

beta0\_LMS(i,j,k) = Y(j)+Y(k)-beta1\_LMS(i,j,k)\*(X\_2(j)+X\_2(k));

d(i,j,k) = median((Y-(beta0\_LMS(i,j,k)+beta1\_LMS(i,j,k)\*X\_2)).^2);

end

end

end

for i=1:n

for j=1:n

for k=1:n

if d\_cap>d(i,j,k);

d\_cap=d(i,j,k);

i\_cap = i;

j\_cap = j;

k\_cap = k;

end

end

end

end

b0\_LMS = beta0\_LMS(i\_cap,j\_cap,k\_cap);

b1\_LMS = beta1\_LMS(i\_cap,j\_cap,k\_cap);

% LMS estimation on Bootstrap Samples

d\_cap\_b = Inf\*ones(1,B);

X\_2\_sorted = sort(X\_2);

beta0\_LMS\_b = zeros(n,n,n,B);

beta1\_LMS\_b = zeros(n,n,n,B);

d = zeros(n,n,n,B);

for l = 1:B

for i = 1:n

for j = 1:n

for k=1:n

beta1\_LMS\_b(i,j,k,l) = (Y\_b(i,l)-Y\_b(k,l))/(X\_2\_b(i,l)-X\_2\_b(k,l));

beta0\_LMS\_b(i,j,k,l) = Y\_b(j,l)+Y\_b(k,l)-beta1\_LMS\_b(i,j,k,l)\*(X\_2\_b(j,l)+X\_2\_b(k,l));

d(i,j,k,l) = median((Y\_b(:,l)-(beta0\_LMS\_b(i,j,k,l)+beta1\_LMS\_b(i,j,k,l)\*X\_2\_b(:,l))).^2);

end

end

end

end

i\_cap\_b = zeros(1,B);

j\_cap\_b = zeros(1,B);

k\_cap\_b = zeros(1,B);

for l=1:B

for i=1:n

for j=1:n

for k=1:n

if d\_cap\_b(l)>d(i,j,k,l);

d\_cap\_b(l) = d(i,j,k,l);

i\_cap\_b(l) = i;

j\_cap\_b(l) = j;

k\_cap\_b(l) = k;

end

end

end

end

end

b0\_LMS\_b = zeros(1,B);

b1\_LMS\_b = zeros(1,B);

for l=1:B

b0\_LMS\_b(l) = beta0\_LMS\_b(i\_cap\_b(l),j\_cap\_b(l),k\_cap\_b(l),l);

b1\_LMS\_b(l) = beta1\_LMS\_b(i\_cap\_b(l),j\_cap\_b(l),k\_cap\_b(l),l);

end

% Analysis on Beta1 estimate by LMS

min\_b1\_LMS\_b = min(b1\_LMS\_b(1,:));

max\_b1\_LMS\_b = max(b1\_LMS\_b(1,:));

mean\_b1\_LMS\_b = mean(b1\_LMS\_b(1,:));

median\_b1\_LMS\_b = median(b1\_LMS\_b(1,:));

std\_b1\_LMS\_b = std(b1\_LMS\_b(1,:));

% Confidence Intervals for beta1 using LMS estimated beta1

sigma\_cap\_LMS = sqrt((sum((Y-(b0\_LMS+b1\_LMS\*X\_2)).^2))/(n-2));

sigma\_cap\_LMS\_b = zeros(1,B);

theta\_star\_LMS\_b = zeros(1,B);

for j=1:B

sigma\_cap\_LMS\_b(j) = sqrt((sum((Y\_b(:,j)-(b0\_LMS+b1\_LMS\*X\_2\_b(:,j))).^2))/(n-2));

theta\_star\_LMS\_b(j) = (b1\_LMS\_b(j)-(b1\_LMS))/((sigma\_cap\_LMS\_b(j))/(sqrt(sum((X\_2\_b(:,j)-mean(X\_2\_b(:,j))).^2))));

end

theta\_star\_LMS\_b\_sorted = sort(theta\_star\_LMS\_b);

omega\_LMS = theta\_star\_LMS\_b\_sorted((1-(alpha/2))\*B);

% In original sample

CI\_L\_beta1\_LMS = b1\_LMS-((omega\_LMS\*sigma\_cap\_LMS)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

CI\_U\_beta1\_LMS = b1\_LMS+((omega\_LMS\*sigma\_cap\_LMS)/(sqrt(sum((X\_2-mean(X\_2)).^2))));

% In Bootstrap samples

CI\_L\_beta1\_LMS\_b = zeros(1,B);

CI\_U\_beta1\_LMS\_b = zeros(1,B);

for j=1:B

CI\_L\_beta1\_LMS\_b(j) = b1\_LMS\_b(1,j)-((omega\_LMS\*sigma\_cap\_LMS\_b(j))./(sqrt(sum((X\_2\_b(:,j)-mean(X\_2\_b(:,j))).^2))));

CI\_U\_beta1\_LMS\_b(j) = b1\_LMS\_b(1,j)+((omega\_LMS\*sigma\_cap\_LMS\_b(j))./(sqrt(sum((X\_2\_b(:,j)-mean(X\_2\_b(:,j))).^2))));

end

% Analysis on Confidence Intervals for beta1 estimated by LMS

% CI Lower Limit

min\_CI\_L\_beta1\_LMS\_b = min(CI\_L\_beta1\_LMS\_b);

max\_CI\_L\_beta1\_LMS\_b = max(CI\_L\_beta1\_LMS\_b);

mean\_CI\_L\_beta1\_LMS\_b = mean(CI\_L\_beta1\_LMS\_b);

median\_CI\_L\_beta1\_LMS\_b = median(CI\_L\_beta1\_LMS\_b);

std\_CI\_L\_beta1\_LMS\_b = std(CI\_L\_beta1\_LMS\_b);

% CI Upper Limit

min\_CI\_U\_beta1\_LMS\_b = min(CI\_U\_beta1\_LMS\_b);

max\_CI\_U\_beta1\_LMS\_b = max(CI\_U\_beta1\_LMS\_b);

mean\_CI\_U\_beta1\_LMS\_b = mean(CI\_U\_beta1\_LMS\_b);

median\_CI\_U\_beta1\_LMS\_b = median(CI\_U\_beta1\_LMS\_b);

std\_CI\_U\_beta1\_LMS\_b = std(CI\_U\_beta1\_LMS\_b);

% Bootstrap sample number for which beta1 lies in the confidence limits by LMS

diff\_LMS = Inf\*ones(1,B);

for j=1:B

if beta1<=CI\_U\_beta1\_LMS\_b(j) && beta1>=CI\_L\_beta1\_LMS\_b(j)

diff\_LMS(j) = abs(((CI\_L\_beta1\_LMS\_b(j)+CI\_U\_beta1\_LMS\_b(j))/2)-beta1);

end

end

for j=1:B

if diff\_LMS(j)==min(diff\_LMS)

j\_min=j;

end

end

beta1\_CI\_check\_LMS = j\_min;

% generating table in excel

filename = 'table 3.xlsx';

A = {'Beta\_1','statistic','OLS','LAD','LMS';...

'Confidence Interval','Number containing beta1',...

beta1\_CI\_check\_OLS,beta1\_CI\_check\_LAD,beta1\_CI\_check\_LMS;...

'Lower Limit of Confidence Interval',...

'minimum',min\_CI\_L\_beta1\_OLS\_b,min\_CI\_L\_beta1\_LAD\_b,min\_CI\_L\_beta1\_LMS\_b;...

'','maximum',max\_CI\_L\_beta1\_OLS\_b,max\_CI\_L\_beta1\_LAD\_b,max\_CI\_L\_beta1\_LMS\_b;...

'','mean',mean\_CI\_L\_beta1\_OLS\_b,mean\_CI\_L\_beta1\_LAD\_b,mean\_CI\_L\_beta1\_LMS\_b;...

'','median',median\_CI\_L\_beta1\_OLS\_b,median\_CI\_L\_beta1\_LAD\_b,median\_CI\_L\_beta1\_LMS\_b;...

'','standard deviation',std\_CI\_L\_beta1\_OLS\_b,std\_CI\_L\_beta1\_LAD\_b,std\_CI\_L\_beta1\_LMS\_b;

'Upper Limit of Confidence Interval',...

'minimum',min\_CI\_U\_beta1\_OLS\_b,min\_CI\_U\_beta1\_LAD\_b,min\_CI\_L\_beta1\_LMS\_b;...

'','maximum',max\_CI\_U\_beta1\_OLS\_b,max\_CI\_U\_beta1\_LAD\_b,max\_CI\_U\_beta1\_LMS\_b;...

'','mean',mean\_CI\_U\_beta1\_OLS\_b,mean\_CI\_U\_beta1\_LAD\_b,mean\_CI\_U\_beta1\_LMS\_b;...

'','median',median\_CI\_U\_beta1\_OLS\_b,median\_CI\_U\_beta1\_LAD\_b,median\_CI\_U\_beta1\_LMS\_b;...

'','standard deviation',std\_CI\_U\_beta1\_OLS\_b,std\_CI\_U\_beta1\_LAD\_b,std\_CI\_U\_beta1\_LMS\_b;

'Estimated Beta\_1',...

'minimum',min\_b1\_OLS\_b,min\_b1\_LAD\_b,min\_b1\_LMS\_b;...

'','maximum',max\_b1\_OLS\_b,max\_b1\_LAD\_b,max\_b1\_LMS\_b;...

'','mean',mean\_b1\_OLS\_b,mean\_b1\_LAD\_b,mean\_b1\_LMS\_b;...

'','median',median\_b1\_OLS\_b,median\_b1\_LAD\_b,median\_b1\_LMS\_b;...

'','standard deviation',std\_b1\_OLS\_b,std\_b1\_LAD\_b,std\_b1\_LMS\_b;};

sheet = 1;

xlRange = 'A1';

xlswrite(filename,A,sheet,xlRange);